$R = \lambda r_1^{\alpha_1} \cdots r_n^{\alpha_n}$ $\quad S = \mu s_1^{\beta_1} \cdots s_n^{\beta_n}$

eg $r_1 = r_1' d_1$, $s_1 = s_1' d_1$

reduction ~~commune~~ eg $r_1 = r_1' d_1$, $s_1 = s_1' d_1$

$R' = \lambda r_1'^{\alpha_1} d_1^{\alpha_1 - \min(\alpha_1, \beta_1)} \cdots$

$D_1 = \underbrace{d_1^{\min(\alpha_1,\beta_1)}}_{\text{si } \alpha_1 \neq \beta_1} \cdots \qquad D_0 = \underbrace{d_1^{\alpha_1}}_{\text{si } \alpha_1 = \beta_1}$

$S' = \mu s_1'^{\beta_1} d_1^{\beta_1 - \min(\alpha_1,\beta_1)} \cdots$

$R = R'D$, $S = S'D$ $\qquad D = D_0 D_1$

cone(R', S', D) irred

prod ~~(R', S)~~ $R \star S = RS'D$

~~factorisation commune~~ fact com seule si $r_2 = s_2$

$R'' = \lambda r_1^{\alpha_1} r_2^{\alpha_1 - \min(\alpha_1,\beta_1)} \cdots$

$C_1 = \underbrace{r_2^{\min(\alpha_1,\beta_1)}}_{\text{si } \alpha_1 \neq \beta_1} \cdots \qquad C_0 = \underbrace{r_2^{\alpha_1}}_{\text{si } \alpha_1 = \alpha_2} \cdots$

$S'' = \mu s_1^{\beta_1} s_2^{\beta_2 - \min(\alpha_2, \beta_2)} \cdots$

ici $R'' \star S''$ n'est pas irred $\qquad R = R''C$, $S = S''C$

cone(R'', C) et cone(S'', C) irred

$R_+ = \lambda r_1^{\max(\alpha_1, 0)} r_2^{\max(\alpha_1, 0)} \cdots$

$R_- = r_1^{\max(-\alpha_1, 0)} \cdots$

$\left(R = \frac{R_+}{R_-}\right)$

$R + S$: $\quad \frac{R_+}{R_-} + \frac{S_+}{S_-}$

$R_- = R'_- D_0 D_1 \qquad R_+ = R''_+ C_0 C_1$

$S_- = S'_- D_0 D_1 \qquad S_+ = S''_+ C_0 C_1$

$\frac{C_0 C_1}{D_0 D_1} \frac{R''_+ S'_- + S''_+ R'_-}{R'_- S'_-}$

$\frac{C_0 C_1}{D_0 D_1 R'_- S'_-}$ déjà irred ; C_1, D_1, R'_-, S'_- sont irred à $R''_+ S'_- + S''_+ R'_-$

(eg un diviseur de C_1 divise soit S''_+ soit R''_+ (mais pas les 2) et ne divise ni S'_- ni R'_-)

Reste à réduire $\frac{C_0}{D_0}$ avec $R''_+ S'_- + S''_+ R'_-$ (si $\neq 0$)

(ie effectuer $p = \frac{C_0}{D_0} \star \left(R''_+ S'_- + S''_+ R'_-\right)_{F, red_2}$) puis cone$\left(p, \underbrace{\frac{C_1}{D_1 R'_- S'_-}}\right)$

$\frac{C_0 C_1}{R'_- S_-} \quad \frac{D_0}{C_0}$

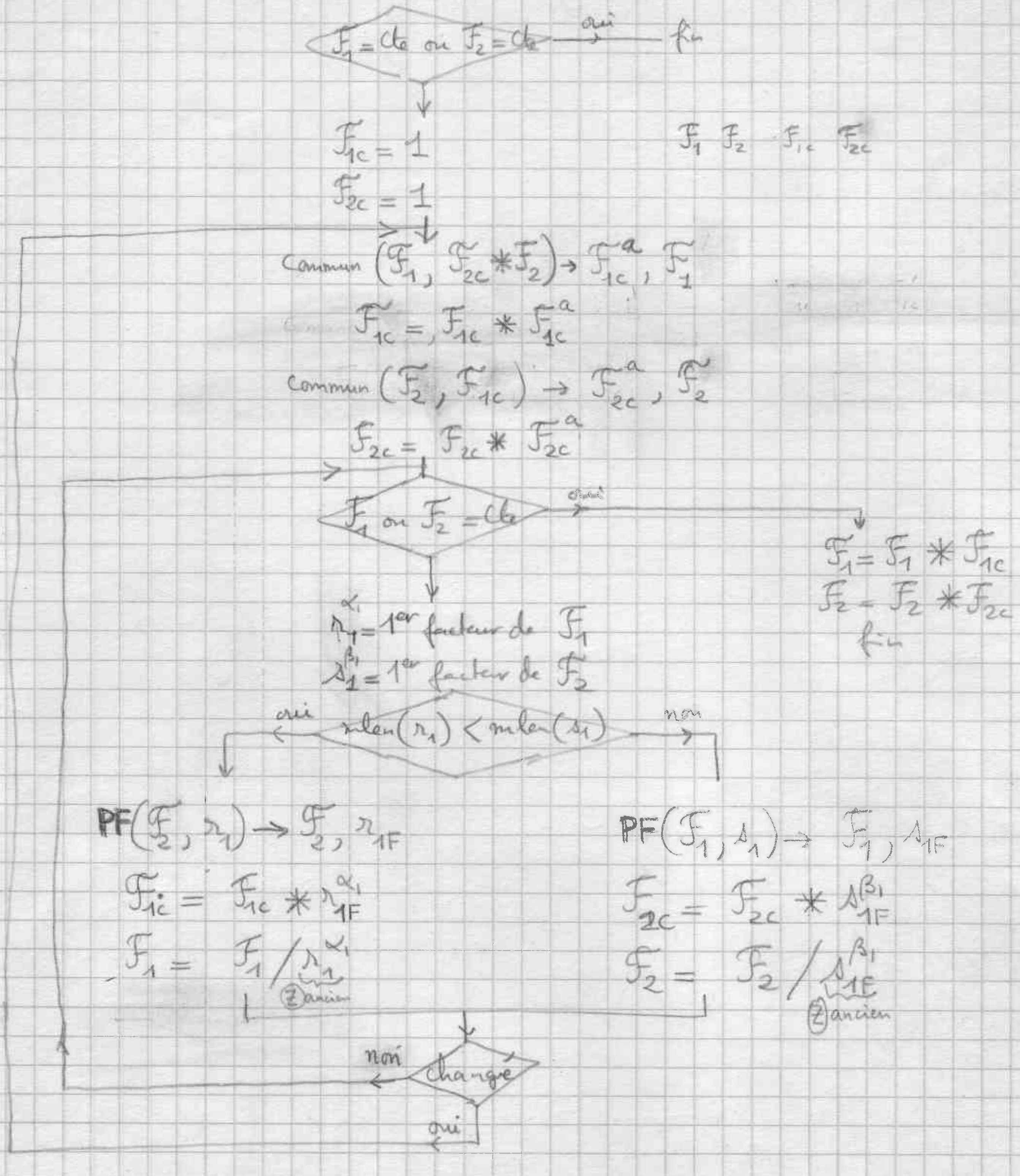
r_1^{α}

$r_1^{\beta} \qquad \beta > \alpha$

r_1^{α}

XREDFF$(\mathcal{F}_1, \mathcal{F}_2)$ remplace par plus factorisé $\mathcal{F}_1 = \lambda f_1^{\alpha_1} \cdots f_m^{\alpha_m}$ $f_1 \cdots f_m$ irréductibles

$\mathcal{F}_2 = \mu f_1^{\beta_1} \cdots f_m^{\beta_m}$



~~XRED~~

PF $(\mathcal{F}_1, s), \alpha$

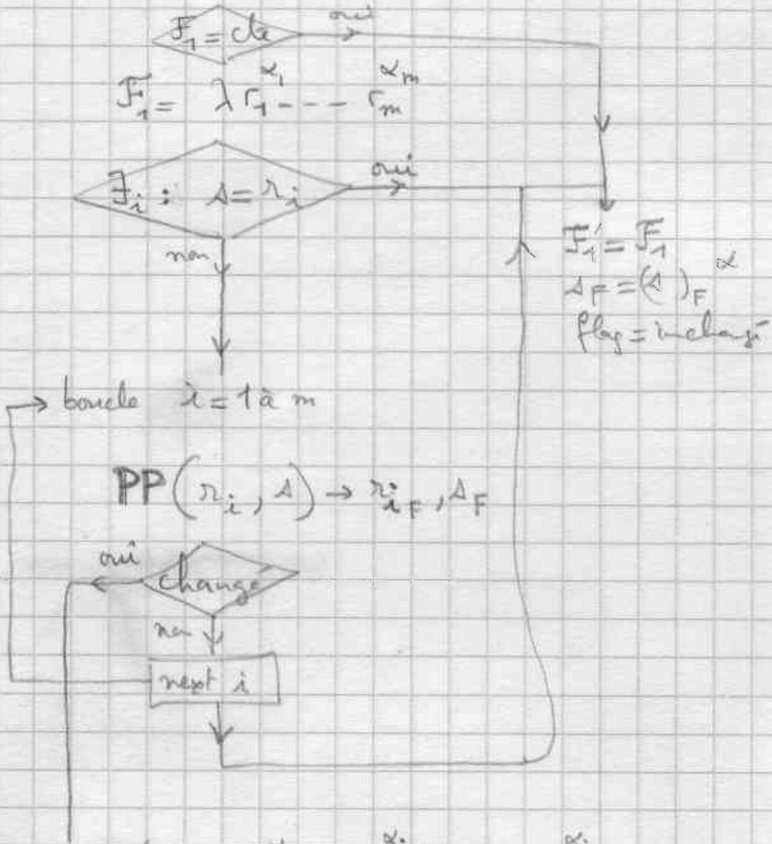
A0 A1 D0

soit $\mathcal{F}_1' = \lambda f_1^{\alpha_1} \cdots f_m^{\alpha_m}$ $f_1 \cdots f_m$ premiers entre-eux

$s_F = \mu f_1^{\beta_1} \cdots f_m^{\beta_m}$

flag = $\begin{cases} \text{changé} \\ \text{inchangé} \end{cases}$

PF



$\mathcal{F}_1' = \lambda r_1^{\alpha_1} \cdots r_{i-1}^{\alpha_{i-1}} * r_{iF}^{\alpha_i}$

$\mathcal{F}_1'' = r_{i+1}^{\alpha_{i+1}} \cdots r_m^{\alpha_m}$

commun $(s_F, r_{iF}) \to s_{F_c}, s_{F_d}$ $\quad (s_F = \underbrace{s_{F_c}}_{\text{facteurs communs avec } r_{iF}} * s_{F_d})$

$\mathbf{FF}(\mathcal{F}_1'', s_{F_d}) \to \Phi, S$

$\mathcal{F}_1' = \mathcal{F}_1' * \Phi$

$s_F = (s_{F_c} * S)^{\alpha}$

flag = changé

$\mathcal{F}_1' \quad s_{F_c} \quad \underbrace{s_{F_d}}_{S} \quad \underbrace{\mathcal{F}_1''}_{\Phi}$

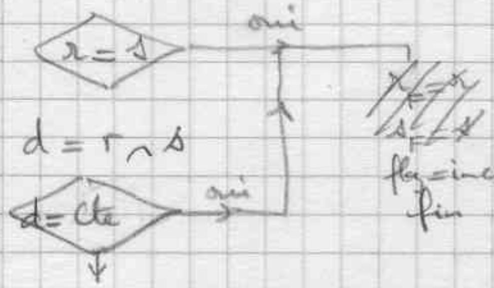
XRED —

PP(r, s) soit $r_F = \lambda f_1^{\alpha_1} \cdots f_m^{\alpha_m}$ $f_1 \cdots f_m$ premiers entre eux

$s_F = \mu f_1^{\beta_1} \cdots f_m^{\beta_m}$

flag = $\begin{cases} \text{changé (si nouveaux facteurs)} \longrightarrow \overset{A2}{\boxed{r_F}}\ \overset{A3}{\boxed{s_F}} \quad \text{conserve A0/A1} \\ \text{inchangé} \xrightarrow{\text{inchangé}} \emptyset \end{cases}$

PP



$r' = r/d$

$PP(d, r') \to d_F, r'_F$

$PF(d_F, \frac{s}{d}) \to D_F, S'_F$

$r_F = \text{conc}(D_F, r'_F)$

$s_F = \text{conc}(D_F, S'_F)$

flag = changé